

Limits on the Weyl meson parameters due to Fermi-LAT gamma-ray observations

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Abstract

We use gamma-ray observations by the Fermi Large Area Telescope (Fermi-LAT) to impose limits on the properties of the hypothetical particle Weyl meson. Such mesons arise in theories which display local scale invariance and act as dark matter candidate. In a generalized locally scale invariant Standard Model it can decay or annihilate to the photon pair through the Higgs channel. We find that Fermi-LAT observations severely constrain the parameters of the Weyl meson.

Keywords: Scale invariance, Weyl meson, Dark matter, Fermi-LAT

1. Introduction

In recent years the hunt for the existence of dark matter (DM) in the Universe has increased significantly. There are many groups looking for the signature of dark matter existence in the Universe either directly or indirectly[1, 2, 3, 4, 5, 6, 7]. There also exist many theoretical models for the dark matter particles[8, 9, 10, 11, 12]. The Weyl meson, which is a gauge particle of the local scale symmetry introduced by Weyl[13] in 1929, is also considered as a potential Dark matter particle. The local scale invariant theories have many cosmological and particle physics applications and are well studied in the literature [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. The mechanism for breaking this symmetry has been discussed in the Ref.[29, 30, 31]. However, in the model containing only one Higgs doublet, the Higgs particle gets eliminated from the particle spectrum [32, 33]. As now the Higgs particle has been found in the Large Hadron Collider (LHC) and is also required to make the theory perturbatively reliable[34, 35], the local scale invariant theories need to be modified to incorporate the Higgs particle. A generalized locally scale invariant theory including extra one scalar field has been discussed in the Ref.[36, 37]. In the model discussed in Ref.[36], the Higgs particle remains in the physical particle spectrum and also the Weyl meson gets weakly coupled to the Higgs boson. The Weyl-Higgs coupling may leads to many cosmological and collider implications, as shown in Ref.[38].

Till now there are no conclusive observations to infer the dark matter presence in the Universe. As it does not interact, or may interact very weakly, with SM particles, it is very difficult to find any direct evidence of the dark matter particles. The possible indirect evidence of dark matter presence in the Universe may be the observation of annihilation or decay product of these particles. It was shown in Refs.[39] that the annihilation of dark matter particles into photon pair can produce the monochromatic gamma rays. Hence a clear peak, in otherwise continuous astrophysical background flux, could be a smoking gun signature for the dark matter existence in the Universe[40, 41, 42, 43]. Recently several groups[7, 44, 45] have noted a similar type of spectral line at 130GeV of energy in publicly available data from the Fermi Large Area Telescope(Fermi-LAT)[46]. They consider the annihilation process of dark matter and find the corresponding cross-section, $\langle \sigma v \rangle \approx 2.27 \pm 0.57^{+0.32}_{-0.51} \times 10^{-27} \text{cm}^3 \text{s}^{-1} (1.27 \pm 0.32^{+0.18}_{-0.28} \times 10^{-27} \text{cm}^3 \text{s}^{-1})$ when using the

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NFW (Einasto) dark matter profile[44] in order to explain the observed flux. The statistical significance for the line structure is 4.6σ , or 3.2σ after the trial factor correction[44]. This claim was followed and opposed by many groups[47, 48, 49, 50]. Recent update addressing this issue is given in the Ref.[51].

In this work we would not address the conflicts about the origin of this gamma-ray feature, but instead analyze the probability of the Weyl meson to produce this type of spectrum. The detailed phenomenological implications of the Weyl meson has already been discussed in the Ref.[38]. Here we first calculate the annihilation cross-section of the Weyl meson to photon pair and find out the coupling parameter for which the observed flux can be explained. Next we calculate the gamma ray flux from the decay of the Weyl meson and compare it with the background gamma-ray flux.

This paper is organised as following. In Sec. 2 we summarize the generalized local scale invariant Standard Model. In Sec. 3 we consider the Weyl meson annihilation as the source of 130GeV gamma-ray line and find the corresponding value of coupling parameter λ_H and λ_S . In Sec. 4 we calculate the gamma-ray flux from the decay of Weyl meson and determine the range of parameters within the observational limit. Finally we conclude and discuss our result in Sec. 5.

2. Generalized local scale invariant theory with one scalar field revisited

We start with the generalized action for local scale invariant Standard Model (SM) given in Ref.[36, 52]

$$\mathcal{S} = \int \sqrt{-\bar{g}} \left(\left[\frac{\beta}{8} \chi^2 + \frac{\beta_1}{4} \mathcal{H}^\dagger \mathcal{H} \right] \bar{R}' + \bar{g}^{\mu\nu} (D_\mu \mathcal{H})^\dagger (D_\nu \mathcal{H}) + \frac{1}{2} \bar{g}^{\mu\nu} (D_\mu \chi) (D_\nu \chi) - \frac{1}{4} \lambda \chi^4 - \frac{1}{4} \lambda_1 [2\mathcal{H}^\dagger \mathcal{H} - \lambda_2 \chi^2]^2 \right) \quad (1)$$

The couplings β and λ_1 are chosen of the order of unity and the couplings $\lambda, \lambda_2 \ll 1$, such that $\lambda \sim H_0^2/M_{\text{PL}}^2$ and $\lambda_2 \sim v^2/M_{\text{PL}}^2$.

Here \mathcal{H} is the Higgs doublet denoted as $\mathcal{H} = \mathcal{H}_0 + \hat{\mathcal{H}}$ and $\mathcal{H}_0, \hat{\mathcal{H}}$ are given as

$$\mathcal{H}_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \hat{\mathcal{H}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi_3 \end{pmatrix}. \quad (2)$$

Classically to obtain the gravitational constant we must have

$$(\beta \chi_0^2 + \beta_1 v^2) \approx M_{\text{pl}}^2. \quad (3)$$

The scale-covariant curvature scalar \bar{R}' is defined in Ref.[32, 53] and D_μ is the scale-covariant derivative, $D_\mu = \partial_\mu - f S_\mu$, where f is the gauge coupling constant and S_μ is the Weyl meson field. By making the quantum expansion of the fields around its classical values,

$$\chi = \chi_0 + \hat{\chi}, \quad \phi_3 = \phi_{3,0} + \hat{\phi}_3, \quad \bar{g}_{\mu\nu} = g_{\mu\nu} + h'_{\mu\nu}, \quad (4)$$

we can extract the quadratic mass and mixing terms of the fields. The classical value of the field χ is taken of the order of Planck's mass and the graviton field is redefined,

$$h'_\alpha{}^\beta = \frac{4}{\sqrt{\beta \chi_0^2 + \beta_1 v^2}} h_\alpha{}^\beta, \quad (5)$$

so that its kinetic energy term gets properly normalized. Detailed quantum treatment of this model has been discussed in Ref.[36]. There it was shown that after choosing a proper gauge one can eliminate the mixing terms and identify the mass term of Weyl meson as,

$$M_S^2 = f^2 \left[\chi_0^2 \left(1 + \frac{3\beta}{2} \right) + v^2 \left(1 + \frac{3\beta_1}{2} \right) \right]. \quad (6)$$

Remaining terms of the action can be arranged in a matrix form as $\Phi^T M^2 \Phi/2$, where

$$\Phi = \begin{pmatrix} \tilde{\chi} \\ \phi_3 \\ h \end{pmatrix}, \quad (7)$$

is a scalar field triplet and M is the mass matrix, which may be decomposed into unperturbed and perturbed parts as,

$$M^2 = M_0^2 + \Delta M^2. \quad (8)$$

The diagonalization of unperturbed mass matrix, considering only leading order terms, gives three eigenvalues which can be identified with the mass of Goldstone type mode, Higgs particle and graviton, with eigen functions as $\tilde{\chi}$, ϕ_3 and \tilde{g} , respectively. We can write the action in term of these particles by inverse transformation. The Weyl-Higgs coupling arises from the kinetic energy term of Higgs boson and coupling of Higgs with gravity in the action. So we can write the interaction Lagrangian as [36],

$$\mathcal{L}_{int} = \lambda_H \tilde{\phi}_3^2 S_{;\mu}^\mu + \lambda_S v \tilde{\phi}_3 S^\mu S_\mu + \frac{\lambda_S}{2} \tilde{\phi}_3^2 S^\mu S_\mu + \dots, \quad (9)$$

where we have defined the couplings as,

$$\lambda_H = \frac{3}{4} f \beta_1, \quad \lambda_S = \frac{3}{2} f^2 \beta_1 \quad (10)$$

A similar type of interaction between the dark matter particle and Higgs boson arises in the extension of SM having extra $U(1)$ gauge symmetry, known as Higgs-portal[54, 55]. In this model $U(1)_X$ gauge boson is the dark matter particle, which interacts with the SM particles through mixing between the complex singlet scalar field and the SM Higgs doublet.

In Ref.[38] the parameter f was assumed to be sufficiently small and β_1 sufficiently large such that λ_H is of order unity. In this case the mass of Weyl meson is small and it may produce observable effects at colliders such as LHC. The parameter f is directly related to the mass of the Weyl meson through the equation(6). The colliders and the cosmological limit of these parameters has been discussed in the Ref.[38].

3. Weyl meson annihilation to photon pair

In generalized locally scale invariant Standard Model, the Weyl meson does not interact directly to any of the Standard Model particles except the Higgs boson[36, 38]. It can annihilate to two photons by a Higgs particle through s-channel. Also being massless particles, photons do not couple to Higgs boson directly. So the $H\gamma\gamma$ vertex can be generated with loops involving W^\pm and charged fermions[56]. The dominant Feynman diagrams contributing to this process are as shown in Fig. 1. The SSH and SHH vertex are given in Eq.[9].

Since the final products are massless, we can write the cross section for this process as

$$\langle \sigma v \rangle = \frac{1}{16\pi s} \sum_{spin} |\mathcal{M}_{SS \rightarrow \gamma\gamma}|^2 \quad (11)$$

where \sqrt{s} is the center of mass energy, and

$$\mathcal{M}_{SS \rightarrow \gamma\gamma} = \lambda_S v \epsilon_\mu \epsilon^\mu \left(\frac{i}{s - m_H^2 - i m_H \Gamma_H} \right) \mathcal{M}_{H \rightarrow \gamma\gamma} \quad (12)$$

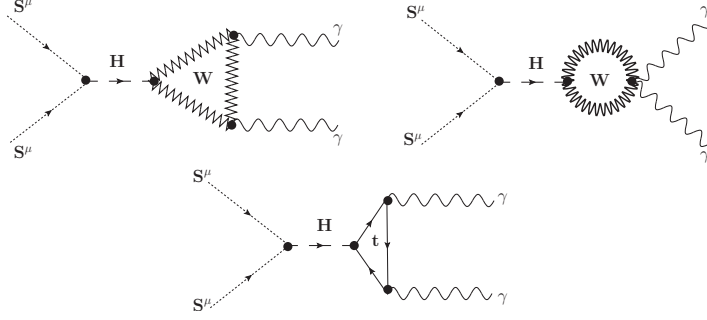
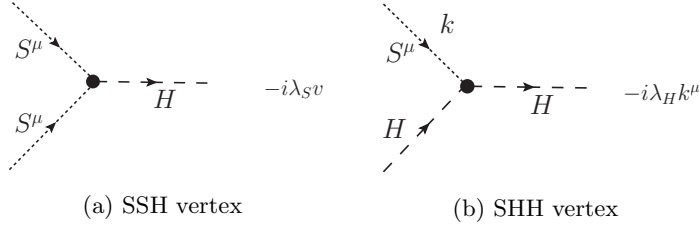


Figure 1: The Feynman diagram for annihilation of the Weyl meson to produce monochromatic gamma ray line.



One can write the $\mathcal{M}_{H \rightarrow \gamma\gamma}$ as follows[56].

$$\begin{aligned}
 \mathcal{M}_{H \rightarrow \gamma\gamma} &= \frac{\alpha_{em} s}{8\pi M_W} \left[\frac{4}{3} F_t + F_W \right] \\
 F_t &= -2\tau[1 + (1 - \tau)f(\tau)], \\
 F_W &= 2 + 3\tau + 3\tau(2 - \tau)f(\tau).
 \end{aligned}$$

Here, $\tau = 4M_i^2/s$ with $i = t, W$ and

$$f(\tau) = \begin{cases} \left(\sin^{-1} \sqrt{1/\tau} \right)^2, & \tau \geq 1 \\ -\frac{1}{4} \left(\ln \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi \right)^2 & \tau < 1 \end{cases}$$

Therefore, we obtain the cross section as

$$\begin{aligned}
 \langle \sigma v \rangle &= \frac{1}{16\pi s} \sum_{spin} |\mathcal{M}|^2 \\
 &= \frac{1}{16\pi s} \lambda_S^2 v^2 \left(2 + \frac{1}{M_S^4} \left(\frac{s}{2} - M_S^2 \right)^2 \right) \frac{|\mathcal{M}_{H \rightarrow \gamma\gamma}|^2}{(s - m_H^2)^2 + m_H^2 \Gamma_H^2}
 \end{aligned}
 \tag{14}$$

where $s = 4M_S^2$, is the square of center of mass energy for the non-relativistic case. To produce the 130GeV gamma-ray line by two body annihilation process, the mass of each annihilating particle must be approximately 130GeV . In Ref.[44], Weigner determined that $\langle \sigma v \rangle$ should be approximately $1.27 \times 10^{-27} \text{cm}^3 \text{s}^{-1}$ in order to explain the observed Fermi gamma-ray flux. Using this value of $\langle \sigma v \rangle$ and $M_S = 130\text{GeV}$ we find from Eq. 14 that the value of coupling parameters are, $\lambda_S \approx 2.4$ and $\lambda_H = \lambda_S/2f \approx 1/f$. The parameter f , which is related to the mass of the Weyl meson through Eq.(6), is assumed to be very small, such that the Weyl meson is not a very massive particle. So we obtain a very large value of SHH vertex, $\lambda_H \gg 1$, from the annihilation of the Weyl meson.

When we consider the three Higgs production process at LHC, the Higgs-Weyl coupling is described by SHH type vertex[36, 38]. So the large value of λ_H obtained from the annihilation leads to the large coupling constant, which will make the theory perturbatively unreliable. The appearance of such a large coupling disfavours the small mass range of the Weyl meson, although it is not entirely ruled out. Qualitatively it will predict a relatively large cross-section for three Higgs boson production at LHC. In this paper we do not pursue this possibility further.

4. Gamma ray flux from the Weyl meson decay

Besides the annihilation, the Weyl meson can also decay into a photon pair. We may observe the gamma ray flux from the decay of these heavy particles. We consider the tree level decay process of the Weyl meson, shown in Fig.2, where blob represent the W^\pm and top quark loop. The gamma rays are produced through

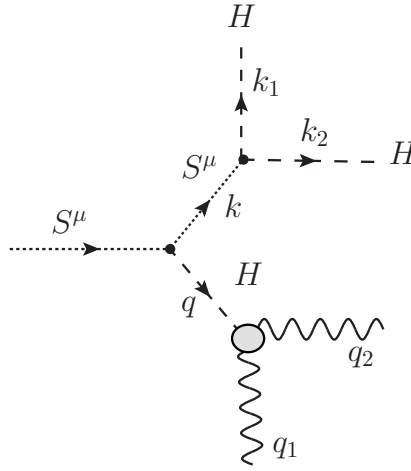


Figure 2: Tree level diagram of the Weyl meson decaying to photon pair.

the Higgs channel. The Higgs boson will emit two photons, each of energy $m_H/2 \approx 63\text{GeV}$, in its rest frame. The higher energy photons can only be produced in the boosted frame of the Higgs boson. But the boosting would make the photon spectrum broad instead of a sharp peak at 130GeV . So the observed peak at 130GeV would not be explained by this process. However, a lower energy peak at 63GeV is expected.

Here we consider the Higgs boson produced with low kinetic energy such that it emits the monochromatic gamma rays of energy 63GeV . We calculate the flux of these 63GeV photons and compare it with the background gamma rays flux in order to impose a limit on the coupling parameter λ_H .

The differential flux of photons from the dark matter decay in the galactic core, observed from a given direction in the sky, is given as [57],

$$\frac{d\Phi_{\text{dec}}}{dE_\gamma d\Omega} = \frac{\Gamma}{4\pi} r_\odot \frac{\rho_\odot}{M_S} \int_{\text{l.o.s}} ds \frac{1}{r_\odot} \left(\frac{\rho_{\text{dm}}(r)}{\rho_\odot} \right) \frac{dN_{\text{dec}}}{dE_\gamma} \quad (15)$$

where the coordinate r , centered on the Galactic Center, reads $r(s, \psi) = (r_\odot^2 + s^2 - 2r_\odot s \cos \psi)^{1/2}$, $r_\odot = 8.5\text{ kpc}$ is the distance from the galactic center (GC) to the Sun and ψ is the angle between the direction of observation in the sky and the GC. In terms of the galactic latitude b and longitude l , one has $\cos \psi = \cos b \cos l$. The integral $\int_{\text{l.o.s}} ds$ is carried along the line of sight. The DM halo density profile is denoted by $\rho_{\text{dm}}(r)$, $\rho_\odot \approx 0.4\text{GeVcm}^{-3}$ is the local DM halo density.

We consider the decay of the Weyl meson at rest and $\tau = 1/\Gamma$ is the life time of the Weyl meson. In the case of $H \rightarrow \gamma\gamma$, the emitted differential photon energy spectrum is given as $\frac{dN_{\text{dec}}}{dE_\gamma} = 2\delta(E - m_H/2)$. We

replace the Dirac delta function $\delta(E - m_H/2)$ with the Lorentzian Function, such that the gamma rays flux at the peak is given by

$$\frac{d\Phi_{\text{dec}}}{dE_\gamma d\Omega} \Big|_{E_\gamma=63\text{GeV}} = \frac{\Gamma}{4\pi} r_\odot \frac{\rho_\odot}{M_S} \int_{1.0.s} ds \frac{1}{r_\odot} \left(\frac{\rho_{\text{dm}}(r)}{\rho_\odot} \right) \frac{2}{\Gamma_H \pi}. \quad (16)$$

The observed gamma-ray flux and background flux varies with energy as shown in the Fig.[3]. As the

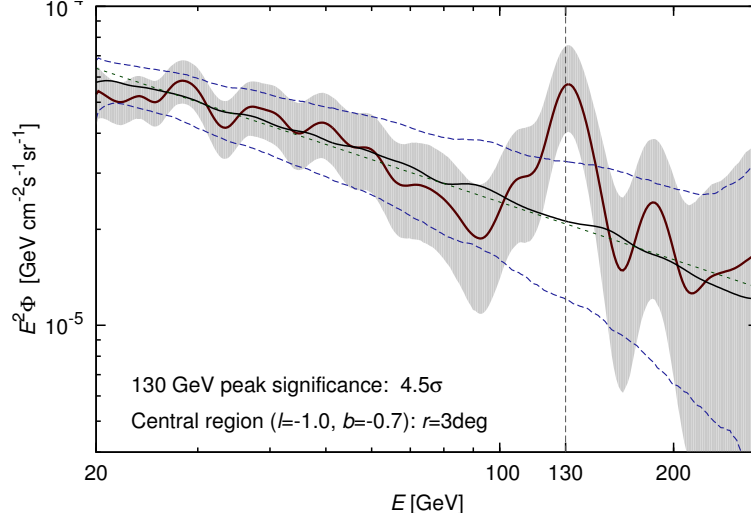


Figure 3: High energy gamma-ray flux together with 95% CL error band as function of photon energy (figure taken from Ref.[45]).

observation does not show any significant peak around 63GeV, so either the flux of these photons is very small compared to the background or the peak lies within the 2σ error band of background flux. From Eq.(16), this observation implies that the decay rate of the Weyl meson follows the constraint

$$\Gamma \leq 4.6 \left(\frac{M_S}{100\text{GeV}} \right) \times 10^{-27} \text{sec}^{-1} \quad \text{or} \quad \tau \gtrsim 10^{26} \text{sec}. \quad (17)$$

This shows that the Weyl meson is a very long lived particle. In our calculations we have used the Einasto DM density profile, given as [44, 58, 59]

$$\rho_{\text{dm}}(r) \propto \exp \left(\frac{-2}{\alpha_E} \frac{r^{\alpha_E}}{r_s^{\alpha_E}} \right), \quad (18)$$

with $\alpha_E = 0.17$ and $r_s = 20$ kpc.

4.1. Calculation of the decay rate of the Weyl meson

In previous section we have obtained the limit on the decay rate of the Weyl meson from the observed and background gamma-ray flux. The decay process of the Weyl meson we are considering here is shown in Fig.2. The amplitude for this process can be written as,

$$\mathcal{M} = \frac{\lambda_H \lambda_S v}{(S_{12} - M_S^2)(S_{34} - m_H^2 - im_H \Gamma_H)} \epsilon^\mu \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_S^2} \right) k^\nu \mathcal{M}_{H \rightarrow \gamma\gamma} \quad (19)$$

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \left| \frac{2 \lambda_H^2 f v}{M_S^2 (S_{34} - m_H^2 - im_H \Gamma_H)} \right|^2 \left(-S_{12} + \frac{(M_S^2 + S_{12} - S_{34})^2}{4M_S^2} \right) |\mathcal{M}_{H \rightarrow \gamma\gamma}|^2.$$

The decay rate of particle of mass M_S is given by,

$$\Gamma_S = \frac{1}{2M_S} \int \sum_{spin} |\mathcal{M}|^2 d\Phi \quad (20)$$

where $d\Phi$ is the phase space integral and for four particle decay process it can be written as [60, 61]

$$\int d\Phi = \int \int \frac{dS_{12}}{2\pi} \frac{dS_{34}}{2\pi} \frac{\beta_{12}}{8\pi} \frac{\beta}{8\pi} \frac{1}{8\pi} \quad (21)$$

with $S_{12} = k^2$, $S_{34} = q^2$ and

$$\beta_{12} = \sqrt{1 - \frac{4m_H^2}{S_{12}}}, \quad \beta = \sqrt{1 - \frac{2(S_{12} + S_{34})}{M_S^2} + \frac{(S_{12} - S_{34})^2}{M_S^4}}. \quad (22)$$

Following the constraint Eq.(17) for the decay rate, Γ , of the Weyl meson, we obtain the limit on the coupling parameter λ_H as a function of mass using Eq.[19,20]. Here we have used $f \approx M_S/m_{pl}$. The resulting parameters are plotted in Fig.4.

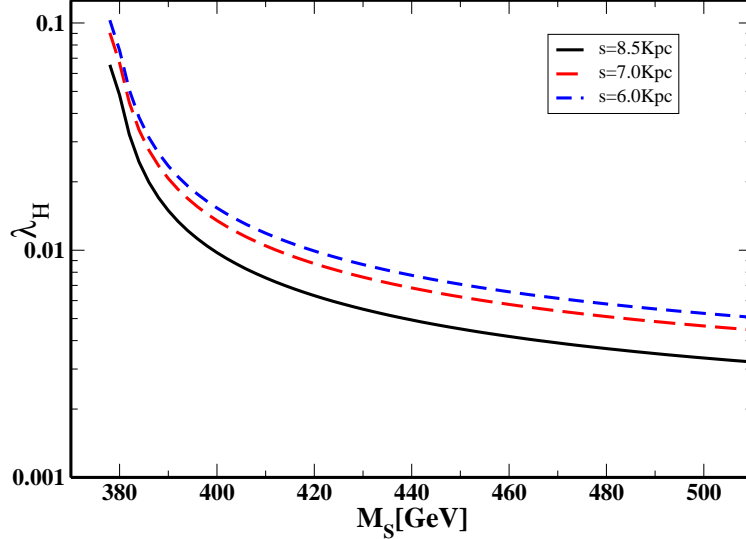


Figure 4: The maximum value of λ_H as the function of the Weyl meson mass for different *l.o.s* distance, s .

In Fig.4, we give result for different values of the line of sight distance, s . The lower mass limit of Weyl meson is due to the fact that here we are considering the decay of the Weyl meson into two real Higgs boson and two gamma-rays such that $M_S \geq 3m_H$. It is clear from the figure that the coupling parameter is very small for all mass range of the weyl meson. For small coupling parameter it may be possible that the Weyl meson may have decoupled from the cosmic soup immediately after the inflation ended or it may not be in equilibrium at all, as discussed in Ref.[38]. So we can not put any constraint on the relic density of the Weyl meson present in the Universe today. But in colliders the small coupling may produce observable effects and we can have the signature of the Weyl meson presence through the three Higgs production [38].

5. Conclusions and Discussions

In this work we have studied the implications of the Weyl meson for the gamma-ray line feature recently observed by the Fermi-LAT. The Weyl meson is a naturally arising dark matter particle in generalized

locally scale invariant Standard Model. The cosmological and colliders implications of the Weyl meson has been discussed earlier. In this work we have studied the range of coupling parameter and mass of the Weyl meson for the recent Fermi-LAT observation of gamma-rays coming from the galactic center. This observation shows a peak at 130GeV which is accounted for by the dark matter annihilation or decay at the center of galaxy. We have found that the annihilation of the Weyl meson of mass 130GeV to produce the required gamma-ray flux need a very large value of coupling parameter λ_H . This make perturbation theory unreliable for small mass range of the Weyl meson in application to collider physics. We also calculated the flux of gamma-ray from the decay of the Weyl meson. The Weyl meson emits two photons through the Higgs channel, so we expect a peak at 63GeV of energy. To emit the photons of higher energy the Higgs boson need to be boosted, which will make the spectrum broad instead of a sharp peak. The Fermi-LAT observations do not show any peak around the 63GeV of energy, which implies that the flux of these gamma rays are small compared to the background gamma-ray flux. Hence we can impose a limit on the parameters using the background gamma-ray flux. We have shown that if the lifetime of the Weyl meson is greater than 10^{26} sec , then the flux of gamma-ray produced is comparable to the background flux and hence the peak may not be seen by the Fermi-LAT. This constraint implies that for all $M_S > 386 \text{ GeV}$, the λ_H parameter is always less than 0.1.

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